Matlab/Freemat: Matrix Eigenvalues and Eigenvectors

That is the eigenvalues and eigenvectors of a matrix A are the non-trivial vectors $\underline{x}$ and scalars λ that satisfy the following equation:

$$A\,\underline{x} = \lambda\,\underline{x}.$$

In Freemat/Matlab, the eigenvalues of the matrix - or the eigenvalues and eigenvectors of a matrix - can be found using the eig command.

Example 1. 2 × 2 matrix

For example the solution of the eigenvalue problem with $A = \begin{pmatrix} 3 & -2 \\ -1 & 4 \end{pmatrix}$ can be determined with the following code.

```
--> A=[3 -2; -1 4]
A =
3 -2
-1  4                              -
-> eig(A)
ans =
2
 5                                 -
-> [E,V]=eig(A)
E =
-0.8944   0.7071
 -0.4472  -0.7071
V =
2 0
 0 5
```

Note that the command eig(A) simply returns the eigenvalues of A, but the command [E,V]=eig(A) returns both the eigenvalues and eigenvectors. The eigenvalues are given by the diagonal components of the matrix V and the corresponding eigenvectors are the columns or the matrix E.

Comparing the results with the document from which the examples came[1], we see that the eigenvalue results match, but the eigenvalue results do not. The eigenvalues found in the referenced document are $\begin{pmatrix} 2 \\ 1 \end{pmatrix}$ and $\begin{pmatrix} 1 \\ -1 \end{pmatrix}$. However, with eigenvectors it is the ratio of the elements that matters, not the actual values. In Matlab/Freemat, the eigenvectors have been normalised[2], so that their norm is one.

Example 2. 3×3 matrix

For example the solution of the eigenvalue problem with $A = \begin{pmatrix} 1 & -1 & 0 \\ 1 & 2 & 1 \\ -2 & 1 & -1 \end{pmatrix}$ can be determined with the following code.

```
--> A=[1 -1 0; 1 2 1; -2 1 -1]
A =
1 -1  0
1  2  1
-2  1 -1
--> eig(A)
ans =
-1.0000
1.0000
2.0000
--> [E,V]=eig(A)
E =
-0.1361   -0.7071   -0.5774              -
0.2722   -0.0000    0.5774
0.9526    0.7071    0.5774
V =
-1.0000        0        0
 0    1.0000        0
 0        0    2.0000
```

Example 3. Generalised Eigenvalue Problem with 2 × 2 Matrices;

Matlab/freemat also have the facility to solve the generalised eigenvalue problem

$$A \underline{x} = \lambda B \underline{x}.$$

For example the following code can be used to solve the generalised eigenvalue problem

$$\begin{pmatrix} 0 & 2 \\ 4 & 2 \end{pmatrix} \underline{x} = \lambda \begin{pmatrix} -2 & 3 \\ 0 & 5 \end{pmatrix} \underline{x}.$$

```
--> A=[0 2; 4 2]
A =
0 2
4 2
--> B=[-2 3; 0 5]
B =
-2  3
0  5
--> eig(A,B)
ans =
0.8000 +  0.4000i
0.8000 -  0.4000i
--> [E,V]=eig(A,B)
E =
0.4037 +  0.5000i    0.4037 -  0.5000i
0.9037 +  0.0963i    0.9037 -  0.0963i
V =
0.8000 +  0.4000i         0
0              0.8000 -  0.4000i
```