

## Matlab/Freemat: Solution of Linear Systems of Equations

A linear system of equations can be written as a matrix-vector equation. Often a linear system of equations has the same number of equations as it has *unknowns*. In this case the matrix will be square and this is the case that is considered in this document.

A linear system of equations can be written in the matrix-vector form

$$A\underline{x} = \underline{b},$$

where  $A$  is a square matrix,  $\underline{b}$  is a given vector and  $\underline{x}$  is the vector of *unknowns*.

The solution can be obtained by inverting  $A$ , and this method will be considered in this document. However, there is also a direct method of *division* in Matlab/Freemat, which is also more computationally efficient than the former method.

### Solution by inverting $A$

By writing the equation in the form

$$\underline{x} = A^{-1}\underline{b},$$

the solution of the linear system of equations can be found by first inverting  $A$  and multiplying the result by  $\underline{b}$ .

For example the equation

$$\begin{pmatrix} 2 & 1 \\ 3 & 2 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 7 \\ 12 \end{pmatrix}$$

may be solved as follows.

```
--> A=[2 1; 3 2]
A =
2 1
3 2
--> b=[7; 12]
b =
7
12
--> inv(A)*b
ans =
2
3
```

### Solution by division

The recommended method is to use the *division* operator ( $\backslash$ ) to solve linear systems of equations. For example the system above can be solved as follows.

```
--> A=[2 1; 3 2]
A =
2 1
3 2
--> b=[7; 12]
b =
7
12
--> A\b
ans =
2.0000
3.0000
```

Given the example of a  $3 \times 3$  system

$$\begin{pmatrix} 1 & 0 & 1 \\ 1 & 2 & 1 \\ 0 & 2 & 1 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix},$$

the solution  $\begin{pmatrix} -1 \\ 0.5 \\ 2 \end{pmatrix}$  can be found in Matlab/Freemat as follows.

```
A =
1 0 1
1 2 1
0 2 1
--> b=[1;2;3]
b =
1
2
3
--> A\b
ans =
-1.0000
0.5000
2.0000
```